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ANALYSIS OF RELATIVISTIC NUCLEUS-NUCLEUS INTERACTIONS
IN EMULSION CHAMBERS

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ABSTRACT

We report on the development of a computer-assisted method for the determination of the angular distribution data for secondary particles produced in relativistic nucleus-nucleus collisions in emulsions. The method is applied to emulsion detectors that were placed in a constant, uniform magnetic field and exposed to beams of 60 and 200 GeV/nucleon ^{16}O ions at the Super Proton Synchrotron (SPS) of the European Center for Nuclear Research (CERN). Linear regression analysis is used to determine the azimuthal and polar emission angles from measured track coordinate data. The software, written in BASIC, is designed to be machine independent, and adaptable to an automated system for acquiring the track coordinates. The fitting algorithm is deterministic, and takes into account the experimental uncertainty in the measured points. Further, a procedure for using the track data to estimate the linear momenta of the charged particles observed in the detectors is included.

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I. INTRODUCTION

In this paper we report on the development of a data analysis method for the rapid determination of the azimuthal and polar emission angles of particles produced in nucleus-nucleus collisions observed in emulsion chambers exposed to relativistic 160 beams. The method makes use of the track coordinate (x,y,z) data that is presently obtained by visual inspection of the developed emulsion plates, using scanning microscopes. Although our initial application focusses on studying charged pions, the method is applicable to data for any emitted particle. The results of this work will be applied to the analysis of heavy ion cosmic ray interactions that are observed in emulsion chambers flown at high altitudes¹. Events from these cosmic ray experiments are especially valuable since they often occur at energies that are substantially greater than those readily achievable with present-day particle accelerators.

The angular distributions of secondary particles, generated in the collision of two nuclei, contain information on the dynamics of the nuclear interaction process. Events that are characterized by large numbers of secondary particles and large transverse momenta are likely candidates to exhibit new fundamental phenomena. One such phenomenon is a new state of matter, the quark-gluon plasma (QGP), that is expected to occur in relativistic collisions that involve unusually high energy densities². Another example rests in the idea that, if the collisions are simple superpositions of proton-like collisions, the produced particles are expected to be emitted isotropically in the center-of-mass frame. In each of these cases, it is very important to examine and understand the angular distributions of particles produced in high energy nuclear interactions, specifically with respect to non-statistical structure that may contain signatures of new physics^{3,4}.

Emulsion chambers are well established as a tool for observing nuclear interactions involving energetic charged projectiles. They have the advantages of being relatively durable and easy to prepare. They can be used to accurately measure the charge and energy of the primary projectile, in addition to the emission angles associated with fragments

and secondary particles produced for the highest energy nuclear interactions. However, being passive detectors, they require a lengthy and somewhat involved set of developing and scanning procedures in order to obtain the raw data needed for analyzing the events they record. Even after the emulsion plates are developed, considerable laboratory work is needed to obtain angular distribution data.

II. OBJECTIVES

The primary objective of this project is to analyze secondary particle distribution data, recorded in emulsions from the EMU05 experiment⁵, for the existence of non-statistical structures. To accomplish this objective, it was necessary to develop appropriate computer software that could be used to find the azimuthal and polar emission angles from particle track coordinate data. The software includes error analysis, and it has been tested successfully with data for which the results are known. In particular, the angular distributions of charged pions, observed in the EMU05 experiment, are to be examined for deviations from isotropy in the center-of-mass frame.

III. EXPERIMENT DESCRIPTION

For the EMU05 experiment, pulsed beams of ^{16}O with energies of 60 and 200 GeV/nucleon were provided by the Super Proton Synchrotron (SPS) at the European Center for Nuclear Research (CERN). The pulse duration was 2s with a total intensity of 3×10^3 ions/cm² pulse. The integrated exposure given to a chamber was 10^4 ions. The beam size was 2.54cmx2.54cm (1 sq. in.) and each chamber was exposed to beam spills shifted laterally from each other by 1 cm. Proportional counter measurements at the chamber, located 30 cm downstream from the beamline end, indicated the beam to be 98% pure.

The chamber used in this work consisted of stacked emulsion plates separated by layers of lead, CR39 plastic and polystyrene. A sketch of the experimental arrangement, showing the approximate dimensions of the chamber, is provided in figure 1. The chamber was placed inside a uniform, 1.8 Tesla magnetic field. A cross sectional view of the detector configuration for which our analysis method was developed, is provided in figure 2. In this case, each emulsion plate had a 70 μm base coated on both sides with 50 μm of emulsion. The separation between the emulsion plates is not constant, but gradually increases in the direction of the beam. This facilitates the measurement of the track curvature, the identification of the charge of the emitted particle, and places an upper limit of 10 GeV on the energy of the secondary particles that can be analyzed. Also, lead plates are placed near the front of the detector where the density of emulsion plates is greater to increase the likelihood of collisions there. This feature also improves the accuracy with which the position of the collision vertex and the track angles can be determined.

IV. DATA REDUCTION METHODS

IV.a. Determination of Emission Angles

The method employed to find the polar and azimuthal emission angles consists essentially of fitting the set of position coordinates, (x_i, y_i, z_i) , for a given track, to the equation of a straight line⁶. The situation is illustrated in figure 3. The vector \mathbf{d} points in the initial direction of motion of the emitted particle. Since the paths are curved, it is recognized from the outset that this approach can be used to obtain a good estimate of the initial direction of motion of the outgoing particle, at the point of collision. Consequently, only those points closest to the collision vertex are used in the calculation.

First, a fit to the line $y = a + bx$ is found using the set of points, (x_i, y_i) , in the x-y plane. The azimuthal angle, ϕ , is then simply obtained from

$$\phi = \tan^{-1}(b) \quad , \quad (1)$$

where b is the slope of the line. The procedure is repeated for the set of points, (r_i, z_i) , in the r-z plane where

$$r_i = \sqrt{(x_i)^2 + (y_i)^2} \quad , \quad (2)$$

and $z = c + mr$. The angle θ is then obtained from

$$\theta = \tan^{-1}(m) \quad . \quad (3)$$

This procedure is performed for each track associated with the event.

Values for b and m are obtained by the minimization of a chi-square quantity given by

$$\chi^2(a,b) = \sum_{i=1}^N \left(\frac{y_i - y(x_i; a, b)}{\sigma_i} \right)^2, \quad (4)$$

where σ_i is the experimental uncertainty in the i th point. The resulting conditions,

$$\frac{\partial \chi^2}{\partial a} = 0$$

and (5)

$$\frac{\partial \chi^2}{\partial b} = 0,$$

must be satisfied, and in doing so yield two equations in two unknowns that are readily solvable for the constants $a(c)$ and $b(m)$. An estimate of the probable uncertainties in the constants can be obtained if the data are treated as independent with each contributing its own bit of uncertainty to the parameters. Consideration of the propagation of errors shows that the variance, σ_f , in the value of any function will be

$$\sigma_f^2 = \sum_{i=1}^N \sigma_i^2 \left(\frac{\partial f}{\partial y_i} \right)^2 \quad (6)$$

where $f = a(c), b(m)$.

If, however, the individual measurement errors of the points σ_i , are not known, then a more accurate estimate of the probable uncertainties in the parameters $a(c)$ and $b(m)$ can be obtained via the following procedure. Set $\sigma_i = 1$ in equations (4), (5), and (6), and multiply the values of σ_f by the additional factor,

$$\sqrt{\chi^2 / (N-2)},$$

where χ^2 is computed by (4). In essence, this latter procedure is equivalent to assuming that one obtains a good fit.

IV.b Calculations of the Linear Momenta

As suggested in the introduction, it is important to identify those events in which a large amount of linear momentum of the incident projectile is transferred to the target nucleus. This may be done by careful examination of the linear momenta reaction products⁷.

The radius of curvature of the path of a secondary particle is related directly to its linear momentum. To show this, consider the motion of a charged particle in a magnetic field. The magnetic force on the particle is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) , \quad (7)$$

where q is the charge on the particle, \mathbf{v} is its velocity, and \mathbf{B} is the magnetic field. In the present case, \mathbf{B} is assumed to be uniform and oriented in the positive y -direction. Thus, the magnitude of the force can be written as

$$F = qvB\sin(\theta') , \quad (8)$$

where θ' is the angle between \mathbf{v} and \mathbf{B} , and the direction of \mathbf{F} is everywhere perpendicular to the plane formed by \mathbf{v} and \mathbf{B} . The curved motion is described in terms of a centripetal acceleration so that,

$$qvB\sin(\theta') = mv^2/R , \quad (9)$$

where m is the mass of the particle and R is its radius of curvature. Since $p = mv$ is the linear momentum of the particle, we have

$$p = qBR\sin\theta' . \quad (10)$$

For convenience, equation (10) may be expressed as⁸

$$p(\text{GeV}/c) = 0.29979 \, q \, B(\text{T}) \, R(\text{cm}) \, \sin\theta' , \quad (11)$$

where q takes on the value ± 1 for pions.

We can derive an estimate of R from the measured track coordinates using the scheme illustrated in figure 4. From the figure,

$$L_i = R \sin \alpha_i \quad (12)$$

where L is the distance along the symmetry axis of the detector, in this case the z -direction, to the i th emulsion plate. Also, note that

$$\Delta x_i = R(1 - \cos \alpha_i) \quad (13)$$

The quantity Δx_i is the perpendicular distance from the beam direction. These last two equations can be combined to give

$$R = \Delta x_i / (1 - \cos(\sin^{-1}(L_i/R))) \quad (14)$$

Since we are interested in obtaining a solution to this last, non-linear equation for R in terms of Δx_i , this may best be done by approximating the $\cos(x)$ and $\sin^{-1}(x)$ functions by their series forms, i.e.,

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + |x| < \infty$$

$$\sin^{-1}(x) = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 \cdot x^5}{2 \cdot 4 \cdot 5} + \dots \quad |x| < 1$$

Using only the first order terms, we obtain

$$R \approx (L_i)^2 / 2\Delta x_i$$

Clearly, this approximation is best suited for measurements involving the coordinates of the first few emulsion plates nearest the interaction vertex, and for reaction products with large p values.

V. SOFTWARE DEVELOPMENT

A computer code that makes use of the analysis methods described in the previous section was written for the Commodore AMIGA⁹ computer. The code is written in BASIC and is designed to be machine independent. It is expected that the code will be executed under the BASIC interpreter supplied with the computer. For input, the program requires files that contain the track coordinate (x,y,z) data that have been obtained for each event by scanning the developed emulsion plates. At present, the program returns the corresponding angles, θ and ϕ for each track, and it also has a provision for estimating the linear momenta of the emitted particles, based upon the track radius of curvature and the magnetic field. Early tests, employing idealized track data, as well as actual track data from a few plates, indicate that the code is operating correctly. A current source listing, to be regarded as preliminary, is provided in Appendix A along with a logic diagram for the code. A detailed description will appear elsewhere, after finalization of the software.

VI. CONCLUSIONS AND RECOMMENDATIONS

Initial development work on the computer software for determining the emission angles and estimating the linear momenta of particles emitted in nucleus-nucleus collisions observed in emulsions has been completed. The software has been tested successfully for correct operation using idealized track data and partial data from the EMU05 experiment. Further testing of the code with complete track data for EMU05 events is recommended to confirm the accuracy of the calculations.

Additional heavy ion experiments involving emulsion chambers of the 5A2 design are planned for the SPS accelerator. The first will employ a ^{32}S beam and is scheduled for September, 1987. Another will use a ^{208}Pb beam that is anticipated being available during the Fall of 1989. Also, the High Energy Astrophysics Branch of SSL has been involved over the past 10 years in a collaborative research program, the Japanese American Collaborative Emulsion Experiment (JACEE), the purpose of which is to study charge particle cosmic ray interactions in emulsion chambers flown at high altitude. To date, seven balloon flights have been conducted and data analysis has been completed for five of these. In view of the large amount of data anticipated to be available from these two efforts, it is recommended that an automated system of coordinate data recording be incorporated with the code development work presently underway in order to reduce the time between plate scanning and final analysis of the angular distributions. Such a system will be especially valuable for analyzing events having high multiplicities.

The present method of calculating the emission angles will work best when data are available for a few closely spaced plates near the interaction vertex of the event. It is therefore of interest to explore alternative means of fitting the track data that make use of functions that better represent the curved path. An initial approach would include using higher order polynomial function approximations to the path, and finding the tangent to the curve at the interaction vertex.

VII. REFERENCES

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4. S. C. McGuire, NASA CR No. 4054, March 1987.
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6. W. H. Press et al., Numerical Recipes, The Art of Scientific Computing, Cambridge University Press, New York, 1986, pp. 504-508.
7. T. H. Burnett et al., Phys. Rev. Lett., 57, 3249 (1986).
8. Private Communication with Y. Takahashi.
9. AMIGA is the Commodore trade name for this computer.

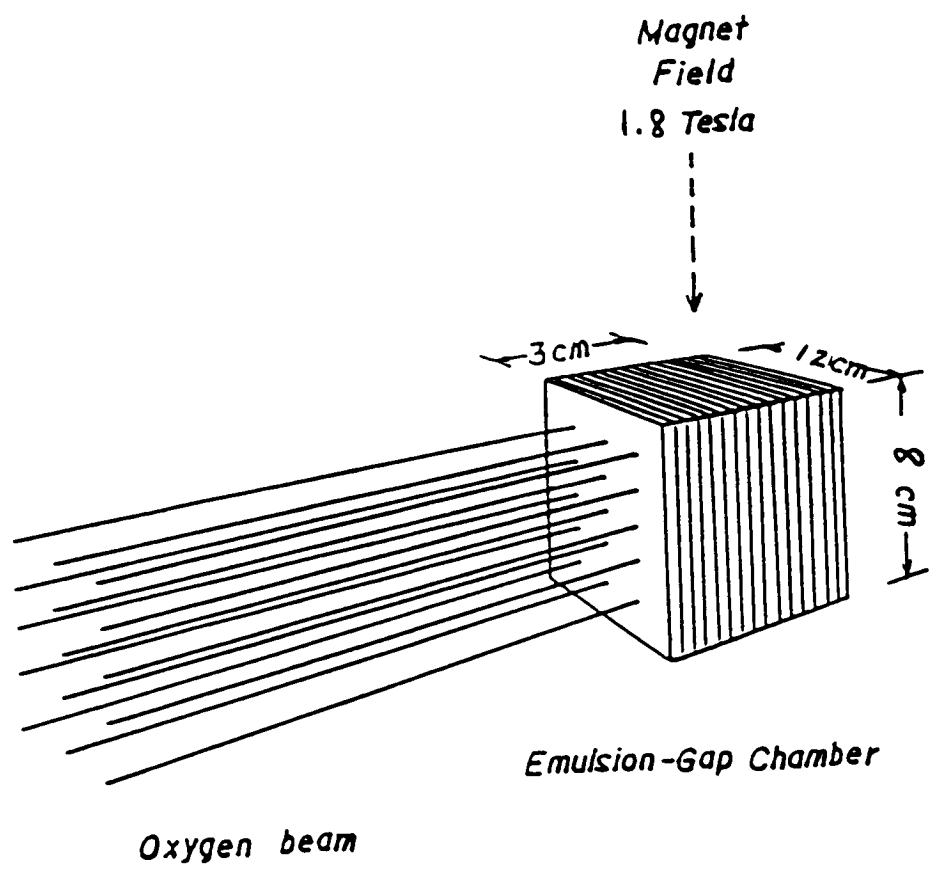


FIGURE 1.

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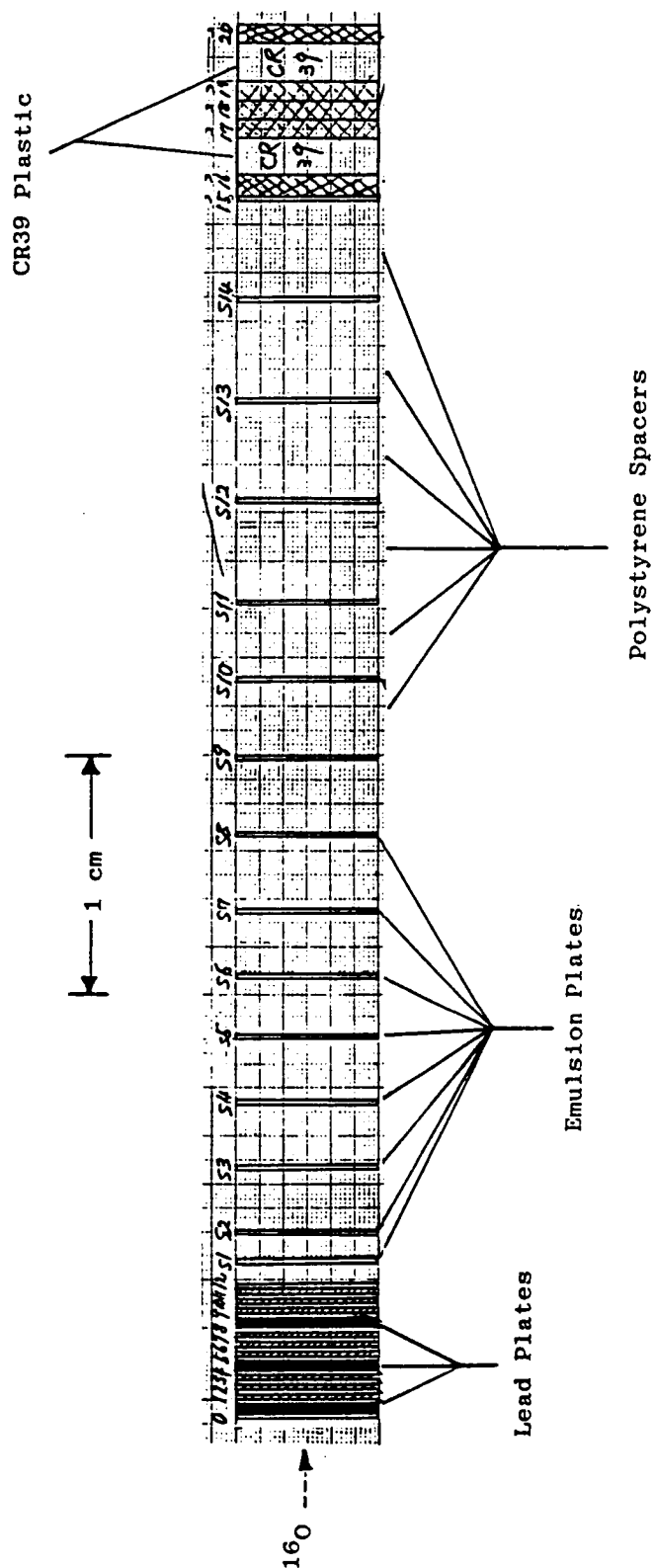


FIGURE 2.

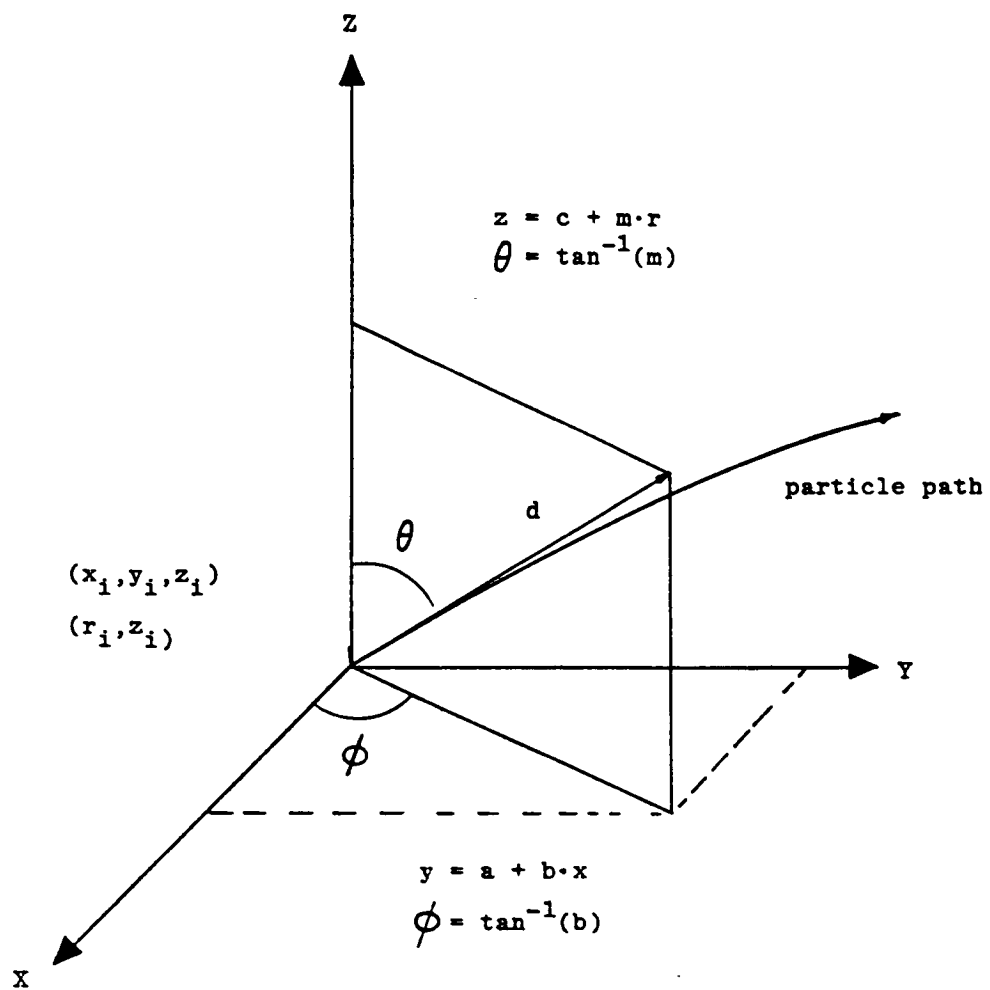


FIGURE 3.

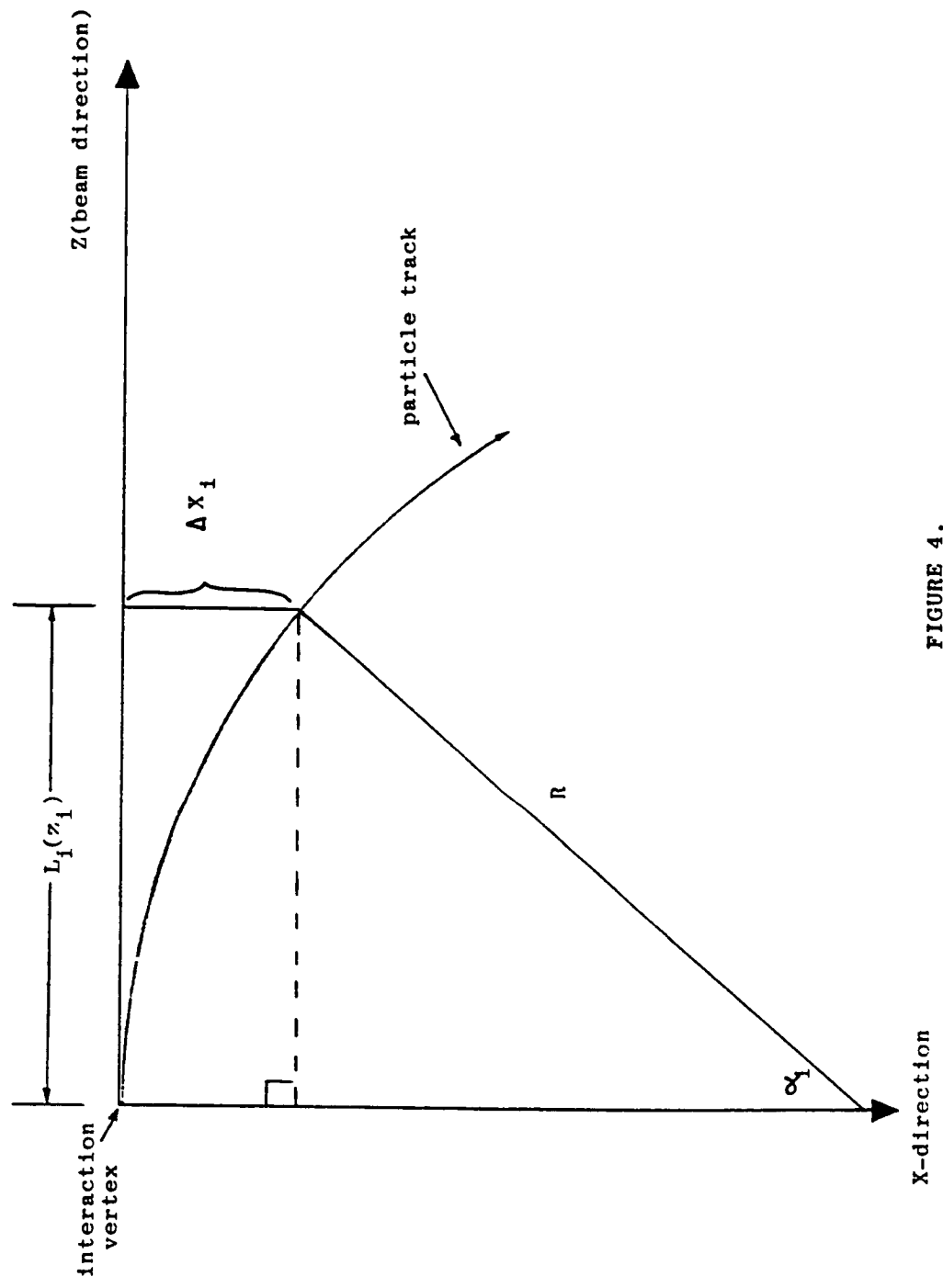
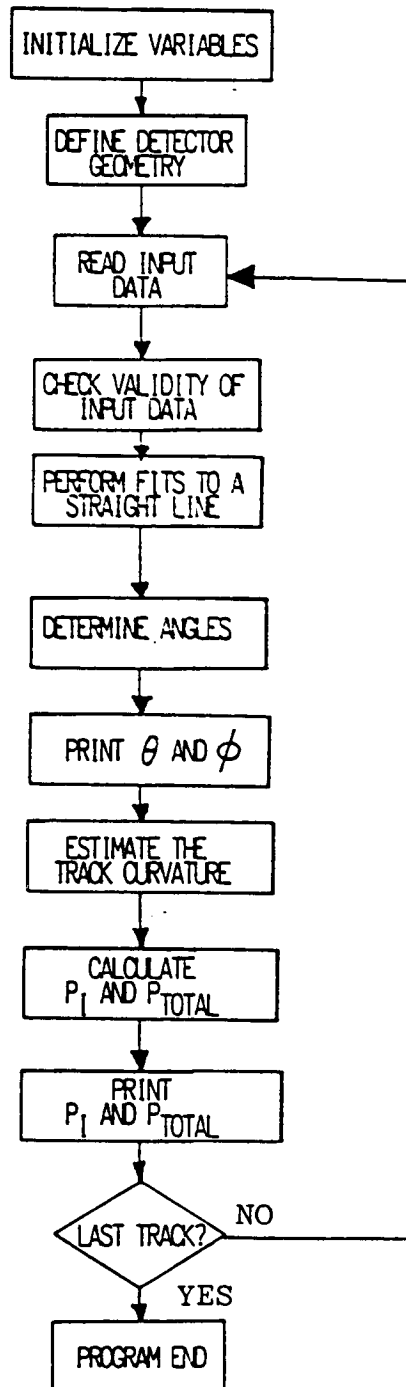


FIGURE 4.

APPENDIX A

Logic Diagram and Source Listing of the Track Coordinate
Analysis Program



```

10 REM****PROGRAM TO CALCULATE THE AZIMUTHAL AND POLAR EMISSION***
20 REM ANGLES FROM THE EMU05 EXPERIMENT DATA.*****
30 REM*****EMU05B.BAS*****
40 DIM A(4),X(50),Y(50),Z(50),SIG(50),R(50)
50 XMAX=80000!:YMAX=80000!:ZMAX=47680!
60 ANGFACT=(180!/3.14159):PFAC%=1
80 CONST = .29979: BFIELD = 1.8
90 QP = 1!: QN = -1!
100 REM*****READ IN THE TRACK DATA.*****
110 REM*****UNITS SHOULD BE MICRONS*****
200 INPUT "FILENAME= ",FILENAM$
210 OPEN FILENAM$ FOR INPUT AS 1
212 LPRINT "DATA FILE = ", FILENAM$
215 REM *****
217 LPRINT
220 REM*****
225 INPUT #1, NRAY%
227 LPRINT " NRAY= ",NRAY%
228 REM*****
230 PPR = 0!: PFL = 0!:PPRTOT=0!:PPLTOT=0!
240 FOR J% = 1 TO NRAY%
245 LPRINT " -----INPUT TRACK DATA FOLLOWS-----"
246 LPRINT
250 NPTS% = 0
260 FOR K% = 1 TO 40
270 NPTS% = NPTS% + 1
275 REM*****
280 INPUT #1, X(K%),Y(K%),Z(K%),SIG(K%)
295 R(K%) = SQR(X(K%)*X(K%) + Y(K%)*Y(K%))
290 LPRINT USING "#####.##"; X(K%),Y(K%),Z(K%),R(K%),SIG(K%)
300 IF X(K%) = -1! THEN NPTS% = NPTS% - 1: GOTO 320
305 REM END INNER LOOP
310 NEXT K%
320 REM*****PERFORM FIT TO A STRAIGHT LINE AND*****
330 REM ***BASED ON THE FITTED DATA , FIND THE EMISSION ANGLES.*****
332 LPRINT " -----EMISSION ANGLES FOLLOW-----"
335 GOSUB 1000
340 REM*****
345 GOTO 510
350 REM*****BASED ON ITS ESTIMATED RADIUS OF CURVATURE, DETERMINE
351 REM THE LINEAR MOMENTUM OF THE TRACK.*****
360 FOR IR% = 1 TO NPTS%
370 RI = Z(IR%)*Z(IR%)
380 RI = RI/(2!*X(IR%))
390 R = R + RI
400 NEXT IR%
410 FLN=NPTS%:AVR = R/FLN:AVR=AVR/10000!
420 PMOM = CONST*QP*BFIELD*AVR
430 PPR = PMOM*SIN(THETA)
440 PFL = PMOM*COS(THETA)
450 PPRTOT = PPRTOT + PPR
460 PPLTOT = PPLTOT + PFL
470 REM*****
480 LPRINT USING "#####.##"; PMOM,PPR,PPRTOT,PPL,PPLTOT
490 REM****
500 LPRINT
510 REM ****GET DATA FOR THE NEXT TRACK, OR
530 REM END OUTER LOOP.*****
540 NEXT J%
545 REM*****
550 CLOSE #1
560 REM*****
570 END

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580 REM
1000 REM***SUBPROGRAM TO PERFORM A BEST FIT TO STRAIGHT LINES;
1005 REM IN THE X-Y AND R-Z PLANES. THESE FITS WILL BE*****
1010 REM** TO DETERMINE THE AZIMUTHAL AND POLAR EMISSION ANGLES*****
1012 REM NOTE:** Y = B*X + A IS THE FORM OF THE STRAIGHT LINE.*****
1015 REM*****FIRST CONSTRUCT THE SUMS OVER THE DATA POINTS
1020 REM NEEDED FOR THE CALCULATION OF THE CONSTANTS.*****
1030 SX=0:SY=0:S=0:SXX=0:SY=0
1045 REM*****USE ONLY A FRACTION OF THE AVAILABLE POINTS.*****
1050 NPTSH% = NPTS%/PFAC%
1060 FOR J2% = 1 TO NPTSH%
1070 SIGSQR = SIG(J2%)*SIG(J2%)
1070 SXY = SXY + X(J2%)*Y(J2%)/SIGSQR
1100 SXX = SXX + X(J2%)*X(J2%)/SIGSQR
1120 SX = SX + X(J2%)/SIGSQR
1130 SY = SY + Y(J2%)/SIGSQR
1150 S = S + 1!/SIGSQR
1160 NEXT J2%
1170 REM*****
1180 REM***THE NEXT STEP IS TO CALCULATE THE VALUES
1190 REM OF THE PARAMETERS A AND B FOR THE AZIMUTHAL ANGLE.*****
1200 DELTA = S*SXX - SX*SX
1210 APhi = (SXX*SY - SX*SXY)/DELTA
1220 BPhi = (S*SXY - SX*SY)/DELTA
1230 REM *****FIND THE UNCERTAINTY IN THE A AND B.*****
1240 SIGMAA = SXX/DELTA:SIGMAA = SQR(SIGMAA)
1250 SIGMAB = S/DELTA:SIGMAB = SQR(SIGMAB)
1260 REM*****
1270 REM*****CALCULATE A CHISQUARE VALUE FOR THE FIT.*****
1280 REM AND RE-ESTIMATE THE UNCERTAINTIES IN A AND B.***
1290 SRES=0!
1300 FOR I% = 1 TO NPTSH%
1310 YP = BPhi*X(I%) + APhi
1320 RESI = (Y(I%) - YP)/SIG(I%)
1330 RESI = RESI*RESI
1340 SRES = SRES + RESI
1350 NEXT I%
1360 CHISQR = SRES
1370 EFAC = CHISQR/(NPTSH% - 2)
1380 EFAC = SQR(EFAC)
1390 REM*****REESTIMATE THE UNCERTAINTY IN THE FITTED CONSTANTS.*****
1400 SIGMAA = SIGMAA*EFAC
1410 SIGMAB = SIGMAB*EFAC
1412 REM*****NEXT, CALCULATE THE ANGLE PHI.*****
1415 PHI = ATN(BPhi):PHI = ANGFACT*PHI
1417 LPRINT "      A      SIGMAA      B      SIGMAB      PHI"
1420 REM*****
1424 IF (X(1) < 0! AND Y(1) < 0!) THEN PHI = PHI + 180!:GOTO 1929
1425 IF X(1) < 0! THEN PHI = PHI + 180!: GOTO 1929
1426 IF Y(1) < 0! THEN PHI = PHI + 360!: GOTO 1929
1429 REM *****
1430 LPRINT USING "*****.###";APhi,SIGMAA,BPhi,SIGMAB,PHI
1440 LPRINT
1500 REM***NOW DO THE SAME FOR THE POLAR ANGLE*****
1505 REM *****X --> R AND Y --> Z.*****
1510 SR=0:SZ=0:SRR=0:SRZ=0:S=0
1505 FOR J1% = 1 TO NPTSH%
1502 SIGSQR = SIG(J1%)*SIG(J1%)
1510 SR = SR + R(J1%)/SIGSQR
1515 SZ = SZ + Z(J1%)/SIGSQR
1520 SRR = SRR + R(J1%)*R(J1%)/SIGSQR
1530 SRZ = SRZ + R(J1%)*Z(J1%)/SIGSQR
1535 S = S + 1!/SIGSQR
1540 NEXT J1%
1542 REM*****FIND THE FITTED CONSTANTS, A AND B FOR THE DETERMINATION OF
1543 REM THE ANGLES.*****

```



```

2045 DELTA = S*SRR - SR*SR
2047 ATHETA = (SRR*SZ - SR*SRZ)/DELTA
2050 BTHETA = (S*SRZ - SR*SZ)/DELTA
2052 REM***NOW FIND THE UNCERTAINTIES IN THE FITTED CONSTANTS.*****
2055 SRES = 0!
2056 SIGMAA = SQR(SRR/DELTA): SIGMAB = SQR(S/DELTA)
2057 FOR IX = 1 TO NPTSH%
2060 ZP = BTHETA*R(IX) + ATHETA
2062 RESI = (Z(IX) - ZP)/SIG(IX)
2065 RESI = RESI*RESI
2067 SRES = SRES + RESI
2068 NEXT IX
2070 CHISQR = SRES
2072 EFACT = CHISQR/(NPTSH% - 2)
2075 EFACT = SQR(EFACT)
2077 SIGMAA = SIGMAA*EFACT
2080 SIGMAB = SIGMAB*EFACT
2081 REM *****NEXT, FIND THE ANGLE THETA.*****
2082 THETA = ATN(1/BTHETA):THETA = ANGFACT*THETA
2083 IF Z(1) < 0! THEN THETA = THETA + 180!
2084 LPRINT "      A      SIGMAA      B      SIGMAB      THETA"
2085 LPRINT USING "*****.##";ATHETA, SIGMAA,BTHETA,SIGMAB,THETA
2086 LPRINT
2090 REM*****
2270 REM *****
2290 RETURN

```